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Limiting Effects of Active Room Compensation using Wave Field Synthesis

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ABSTRACT

Wave field synthesis (WFS) is an auralization technique which allows to control the wave field within the entire listening area. However, reflections in the listening room interfere with the auralized wave field and may impair the spatial reproduction. Active listening room compensation aims at reducing these impairments by using the WFS system for their compensation. Current realizations of WFS systems are limited to the reproduction in a plane only. This reduction in dimensionality leads to effects that limit the performance of active room compensation. This paper analyzes these limiting effects on a theoretical and practical basis.

1. INTRODUCTION

The authentic reproduction of acoustic scenes was and still remains a challenging research topic of the last decades. While many reproduction systems have emerged in the past, the goal of authentic reproduction has only been reached under certain idealistic assumptions. One assumption which is typically made concerns the room where the reproduction takes place (listening room). It is usually assumed that the listening room is almost anechoic. However, due to cost and design considerations listening rooms rarely meet this requirement. Thus, the listening room has to be taken into account when aiming at authentic reproduction.

Figure 1 illustrates the influence of a reverberant listening room on the reproduction by a simplified example. The mapping of an acoustic scene taking place in a church (e.g. a singer performing in the choir) into the listening room is shown exemplarily. The dashed lines in Figure 1 from the virtual source to one exemplary listening position show the acoustic rays for the direct sound and several reflections off the side walls of the recording room. The loudspeaker system in the listening room reproduces the direct sound and reflections in order to create the desired spatial impression of the original scene. The theory behind nearly all of the deployed methods assume an anechoic listening room which does not exhibit any reflections. As a result of a reverberant listening room the loudspeaker wave fields

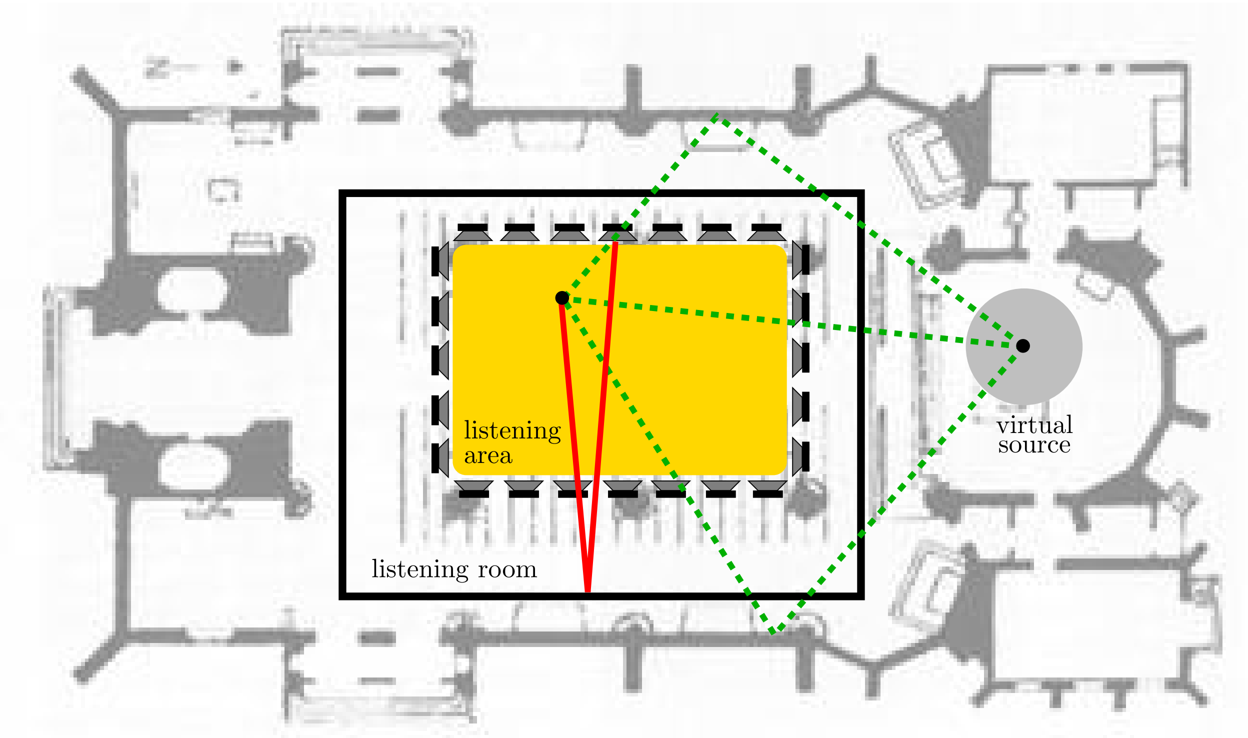


Fig. 1: Simplified example that shows the effect of the listening room on the auralized wave field. The dashed lines from one virtual source to one exemplary listening position show the acoustic rays for the direct sound and two reflections off the side walls of the recording room. The solid line from one loudspeaker to the listening position shows a possible reflection of the loudspeaker wave field off the wall of the listening room.

produce additional reflections in the listening room. The solid line in Figure 1 from one loudspeaker in the upper row to the listening position shows a possible reflection of the loudspeaker wave field off the wall of the listening room. These additional reflections caused by the listening room may impair the desired spatial impression, as this simplified example illustrates. In general, a reverberant listening room will add another room to the desired impression of the recording room. Listening room compensation aims at eliminating or reducing the effect of the listening room on the auralized scene.

A first approach to listening room compensation is to perform damping of the listening room. However, especially for low frequencies damping gets bulky and costly. Besides this passive approach it was also proposed to actively influence the acoustic impedance at the walls of the listening room [1]. Both approaches have in common that they cancel out the reflections for the entire room. They will be termed as *global room compensation* in the following. Most of the currently employed audio reproduction systems somehow limit the area where the listener should stay (listening area). Hence, the compensation of room reflections can also be limited to this area. Consequently, these approaches are termed as *local room compensation* in the following. In the remainder of this work we will only consider local room compensation systems.

Recently it was proposed to use the audio reproduction system to perform active listening room compensation [2, 3, 4, 5]. However, such a system has to fulfill two basic requirements for a successful application of room compensation. The system has to provide

1. control over the wave field within the entire listening area,
2. analysis of the wave field within the entire listening area.

The first requirement ensures that the reflections can be canceled out by destructive interference within the listening area. However, this requires to analyze the wave field within the entire listening area. This is ensured by the second requirement. It was already shown in [2, 3] that wave field synthesis and analysis are suitable techniques for this purpose. Both techniques are derived from basic physical principles. However, in order to arrive at a feasible implementation several simplifications of the original foundations have to be assumed. These simplifications pose limits on the achievable control and analysis capabilities of wave field synthesis and analysis [13, 14, 16]. In the remainder of this paper these effects will be derived and analyzed for wave field synthesis and analysis. The focus of this analysis will be on the limitations these effects pose for active room compensation using circular arrays.

2. SOUND REPRODUCTION

The following section shortly reviews the foundations of sound reproduction systems, the concept of wave field synthesis and its limitations.

2.1. Kirchhoff-Helmholtz Integral

The theoretical basis of sound reproduction is given by the Kirchhoff-Helmholtz integral [6]

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G(\mathbf{x}|\mathbf{x}_S,\omega)\,\frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_S,\omega) - P(\mathbf{x}_S,\omega)\,\frac{\partial}{\partial \mathbf{n}} G(\mathbf{x}|\mathbf{x}_S,\omega) \right) dS \quad (1)$$

where $G(\mathbf{x}|\mathbf{x}_S,\omega)$ denotes a suitable chosen free-field Greens function, $\partial/\partial\mathbf{n}$ the directional gradient and $\mathbf{x}$ a point in the space V ($\mathbf{x} \in V$). The underlying geometry is illustrated in Figure 2. Please note, that the space V may be two- or three-dimensional. In the first case V describes a plane and ∂V the close curve surrounding it, in the second case V describes a volume and ∂V the closed surface surrounding it. The Kirchhoff-Helmholtz integral states that at any listening point within a source-free volume/area the sound pressure can be calculated if both the sound pressure and its gradient are known on the surface/line enclosing the volume. This fundamental principle can be interpreted also as follows: If the sound pressure and its gradient are set appropriately on the surface/line, then the wave field within the volume/area is fully controllable. Thus, an audio reproduction system can be designed on basis of the Kirchhoff-Helmholtz integral. The directional gradient of the acoustic pressure on ∂V can be derived as follows

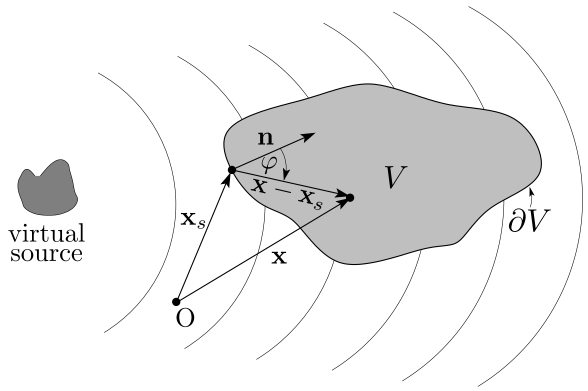


Fig. 2: Parameters used for the Kirchhoff-Helmholtz integral (1).

$$\frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_S,\omega) = -j\omega\rho_0 V_n(\mathbf{x}_S,\omega) \quad (2)$$

where $V_n(\mathbf{x}_S,\omega)$ denotes the particle velocity in normal direction and ρ_0 the static density of air. Thus, the directional gradient of the acoustic pressure is proportional to the particle velocity in normal direction. Up to now the choice of the Greens function was left open. The next two sections will specialize the Kirchhoff-Helmholtz integral to three- and two-dimensional case.

2.2. Three-Dimensional Sound Reproduction

The three-dimensional free-field Greens function is given as [6]

$$G_{3\mathrm{D}}(\mathbf{x}|\mathbf{x}_S,\omega) = \frac{1}{4\pi}\frac{e^{-jk|\mathbf{x}-\mathbf{x}_S|}}{|\mathbf{x}-\mathbf{x}_S|}. \quad (3)$$

Above equation can be interpreted as the field of a point source located at the position $\mathbf{x}_S$. The Kirchhoff-Helmholtz integral (1) also involves the directional gradient of the Greens function. The directional gradient of equation above can be interpreted as the field of a dipole source whose main axis lies in direction of the normal vector $\mathbf{n}$. Thus, the Kirchhoff-Helmholtz integral states in this case, that the acoustic pressure inside the volume V can be controlled by a monopole and a dipole source distribution on the surface ∂V enclosing the volume. These sources will be termed as secondary sources in the following. Outside V the acoustic pressure equals zero.

2.3. Two-Dimensional Sound Reproduction

In general it will not be feasible to control the pressure and its gradient on the entire two-dimensional surface of a three-dimensional volume. Typical reproduction systems are restricted to the reproduction in a plane. This reduction of dimensionality is reasonable for most scenarios due to the spatial characteristics of human hearing [7].
The required reduction in dimensionality for the Kirchhoff-Helmholtz integral (1) is typically performed by assuming that the wave field is independent from the z-coordinate $P(x, y, z, \omega) = P(x, y, \omega)$. The two-dimensional free-field Greens function is then given as [6]

$$G_{2\mathrm{D}}(\mathbf{x}|\mathbf{x}_S, \omega) = \frac{j}{4} H_0^{(2)}(k\,|\mathbf{x} - \mathbf{x}_S|) \qquad (4)$$

where $H_\nu^{(1),(2)}(\cdot)$ denotes the ν-th order Hankel function of first/second kind. Above equation can be interpreted as the field of a monopole line source which intersects the reproduction plane at the position $\mathbf{x}_S$. The directional gradient of Eq. (4) can be interpreted as the field of a dipole line source whose main axis lies in direction of the normal vector $\mathbf{n}$. Thus, the Kirchhoff-Helmholtz integral states in this case, that the acoustic pressure on the plane V can be controlled by a monopole and a dipole line source distribution on the closed curve ∂V surrounding the plane.

2.4. Wave Field Synthesis

The Kirchhoff-Helmholtz integral states that a two-dimensional sound reproduction system can be realized with secondary monopole and dipole line sources. However, in practice its desirable to utilize only one of these source types. The second term in the Kirchhoff-Helmholtz integral (1) involving the dipole sources can be eliminated by choosing the Greens function such, that the directional gradient becomes zero. Additionally it has to be taken care that only those secondary sources are excited where the normal vector $\mathbf{n}$ coincides with the local propagation direction of the wave field to be reproduced [8]. As a result of these modifications, the field outside the area V will not vanish any more.
Since line sources are impractical to realize, the concept of Wave Field Synthesis (WFS) utilizes point sources as secondary sources [9, 10, 11, 12, 13, 14]. Closed loudspeakers constitute reasonable approximations of point sources. Thus, a WFS system can be realized by using loudspeaker arrays located in a plane which surround the listening area. These loudspeakers should be leveled with the listeners ears. The listening area and the surrounding loudspeaker array may have arbitrary shapes. An example for a circular WFS system is shown in Figure 3.
The discretization of the underlying physical and mathematical relations results in spatial aliasing due to spatial sampling. For reproduction purposes this effect does not play a dominant role since the human auditory system doesn't seem to be too sensible for spatial aliasing. A loudspeaker distance of $\Delta x = 10 \ldots 30$ cm has proven to be suitable in practice.

Fig. 3: Circular WFS system with 48 loudspeakers. The loudspeaker array has a radius of $R = 1.50$ m.

2.5. Artifacts of WFS

The theory presented so far states that line sources have to be used as secondary sources for the reproduction in a plane. Since WFS utilizes point sources instead of line sources for the reproduction in a plane, artifacts will occur. The fields of point and line sources are given by Eq. (3) and Eq. (4) respectively. Point sources exhibit an amplitude decay which is inverse proportional to the distance. The amplitude decay of a line source can be derived from the following far-field ($kr \gg 1$) approximations of the Hankel functions [15]

$$H_\nu^{(1)}(kr) \approx \sqrt{\frac{2}{\pi kr}}\; e^{j(kr-\frac{1}{2}\nu\pi-\frac{1}{4}\pi)}, \tag{5a}$$

$$H_\nu^{(2)}(kr) \approx \sqrt{\frac{2}{\pi kr}}\; e^{-j(kr-\frac{1}{2}\nu\pi-\frac{1}{4}\pi)}. \tag{5b}$$

Above approximations state that line sources exhibit an amplitude decay which is inverse proportional to the square root of the distance. Hence, both source types exhibit different amplitude decays over distance. The choice of point sources as secondary sources for WFS results in an amplitude mismatch for the auralized wave field.

Additionally, the reduction to the reproduction in a plane only and the spatial sampling poses limits on the control a WFS system has over the enclosed wave field. The limitation to the reproduction in a plane constricts the suppression of reflections. Reflections emerging from boundaries outside the reproduction plane (elevated reflections) cannot be compensated for the entire listening area. Spatial aliasing limits the frequency up to which a proper control is gained over the wave field. Since active room compensation is built upon destructive interference its application is limited by spatial aliasing. The spatial aliasing frequency for linear loudspeaker arrays is given in [9, 13]. For arbitrary shaped loudspeaker arrays no explicit sampling theorem can be given. In the following it will be assumed that the spatial aliasing condition is reasonable fulfilled.

Summarizing, the control a WFS system provides is limited by the following effects:

1. amplitude mismatch,
2. control only in a plane,
3. spatial aliasing.

The first two artifacts will be discussed in more detail for circular loudspeaker arrays in Section 5. The next section will introduce wave field analysis techniques and their artifacts.

3. WAVE FIELD ANALYSIS

Room compensation requires a thorough analysis of the wave field within the listening area. Wave field analysis (WFA) techniques can be used for this purpose. The following section will introduce the ones used within the context of WFS and room compensation.

3.1. Wave Field Representations

Lets first assume that we have access to the entire two-dimensional pressure field $P(\mathbf{x}, \omega)$. A wave field can be decomposed into the eigensolutions of the wave equation. These eigensolutions are dependent on the particular coordinate system used. Common choices for coordinate systems in three-dimensional space are the Cartesian, spherical and cylindrical coordinate system, and in two-dimensional space the Cartesian and polar coordinate system. The following discussion will be limited to wave field analysis in two dimensions.

The representations of a wave field that are connected to Cartesian and polar coordinates decompose a wave field into plane waves and cylindrical harmonics, respectively [6]. In order to derive these representations it is convenient to change from the Cartesian coordinate system to a polar coordinate system for the spatial coordinates. The decomposition of an acoustic field into plane waves is given as follows [16]

$$P(\alpha, r, \omega) = \frac{|k|}{(2\pi)^2} \int_0^{2\pi} \bar{P}(\theta, \omega)\, e^{-jkr\cos(\theta-\alpha)}\, d\theta \tag{6}$$

where α, r denote the polar representation of the Cartesian coordinates, the acoustic wave number is denoted by $k = \omega/c$ and $\bar{P}(\theta, \omega)$ denotes the plane wave expansion coefficients. The latter can be interpreted as the spectrum of a plane wave with incidence angle θ. The wave field $P(\alpha, r, \omega)$ can be decomposed also into circular harmonics as follows [6]

$$P(\alpha, r, \omega) = \sum_{\nu=-\infty}^{\infty} \breve{P}^{(1)}(\nu, \omega)\, H_\nu^{(1)}(kr)\, e^{j\nu\alpha} + \sum_{\nu=-\infty}^{\infty} \breve{P}^{(2)}(\nu, \omega)\, H_\nu^{(2)}(kr)\, e^{j\nu\alpha} \tag{7}$$

where $\breve{P}^{(1),(2)}$ denote the expansion coefficients in terms of circular harmonics. It can be shown [6] that $\breve{P}^{(1)}$ belongs to an incoming and $\breve{P}^{(2)}$ to an outgoing wave.

In [16] the relation between the expansion coefficients in terms of cylindrical harmonics and plane waves was derived. It is given as the following Fourier series

$$\bar{P}^{(1),(2)}(\theta,\omega) = \frac{4\pi}{|k|} \sum_{\nu=-\infty}^{\infty} j^{\nu} \, \breve{P}^{(1),(2)}(\nu,\omega) \, e^{j\nu\theta} \quad (8)$$

The decomposition into incoming and outgoing waves can be used to distinguish between sources inside and outside the measured area. While sources outside result in an incoming part which is equal to the outgoing part, sources inside the array are only present in the outgoing part.

3.2. Kirchhoff-Helmholtz based Extrapolation

Recording the acoustic pressure $P(x, y, \omega)$ for the entire listening area is not feasible in our context. On the one hand this would require a quite high number of microphones, on the other hand the microphones would occupy the listening positions. A solution to this problem is again provided by the Kirchhoff-Helmholtz integral (1). It states, that the wave field within a plane is given by measuring the acoustic pressure and its directional gradient on the closed curve surrounding the plane. Measuring both quantities allows to decompose the field into an incoming and an outgoing part with respect to the curve surrounding the area. The wave field inside the measured area can then be extrapolated from the boundary measurements using the Kirchhoff-Helmholtz integral (1) together with the two-dimensional Greens function (4). The Greens function in this case acts as a virtual secondary source distribution used for the extrapolation. As shown in Section 2.5, Hankel functions exhibit a far-field amplitude decay witch is inverse proportional to the square root of the distance to the secondary source. Thus, if the wave field of a point source is analyzed, then the extrapolation process will exhibit amplitude errors.

The direct evaluation of the Kirchhoff-Helmholtz integral for a given arbitrary array geometry can be quite complex. For special array geometries (linear, circular) the introduced wave field decompositions provide an elegant solution to this problem [16]. Section 6 will shortly review an efficient algorithm for circular microphone arrays.

3.3. Artifacts of 2D Wave Field Analysis and Extrapolation

As for WFS, the analysis of three-dimensional wave fields using two-dimensional techniques exhibits artifacts. It was shown in the previous section that the extrapolation process from the boundary measurements exhibits amplitude errors for special primary sources. This is due to the amplitude decay of the virtual line sources. Additionally the analysis on the closed curve surrounding the plane cannot fully distinguish between reflections emitting in the analysis plane and elevated reflections. Contributions from elevated reflections will be mixed into the contributions of source located in the analysis plane. As for WFS, the discretization of the underlying physical and mathematical relations results in spatial aliasing due to spatial sampling. In the following it will be assumed that a aliasing condition is reasonable fulfilled.

Summarizing, the analysis capabilities of a WFA system are limited by the following effects:

1. amplitude mismatch of extrapolated field,
2. analysis only in a plane,
3. spatial aliasing.

The first two effects will be discussed in more detail for circular microphone arrays in Section 6.

4. LISTENING ROOM COMPENSATION

The Kirchhoff-Helmholtz integral (1) and its specializations state that the wave field inside a finite space V is fully given by its pressure and/or pressure gradient on the boundary ∂V surrounding the space V. Thus, if the listening room reflections are canceled at the boundary of the listening area, the reproduced wave field within the listening area will be free of undesired reflections.

In the following we will shortly review the room compensation system [3] for WFS. It is based upon the concept of wave domain adaptive filtering (WDAF). Figure 4 shows a block diagram of the room compensation system. The basic idea is to orthogonalize the listening room response $\mathbf{R}$ through the transformations $\mathcal{T}_2$ and $\mathcal{T}_3$. As a consequence, the matrix of compensation filters $\tilde{\mathbf{C}}$ is decomposed into a set of

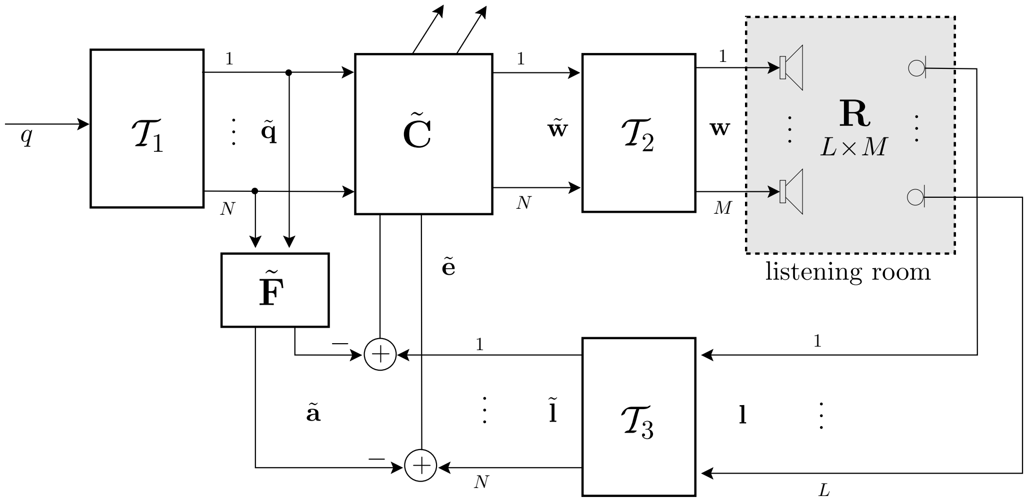


Fig. 4: Block diagram of a room compensation system based on wave domain adaptive filtering.

compensation filters, each acting on only one spatial signal component. The adaption of these compensation filters is then performed independently for each spatially transformed component. The number of compensation filters that have to be adapted is lowered significantly compared to the traditional approaches. Thus the complexity of the filter adaption is reduced. In practice the desired decoupling of the room response can only be reached approximately using data-independent transformations.

The optimal choice of the transformed signal representation depends on the geometry of the problem. It has been shown that circular harmonics provide a suitable basis for circular microphone and loudspeaker arrays [3]. The setup presented in [3] consists of a circular loudspeaker array with a radius of $R = 1.50$ m and 48 loudspeakers (see Figure 3). A microphone array with a radius of $R = 1.00$ m was placed inside the loudpspeaker array in order to analyze the auralized wave field. In the following we will specify the transformations $\mathcal{T}_1$ to $\mathcal{T}_3$ to this particular geometry. The transformation $\mathcal{T}_1$ transforms the virtual source q to be auralized into its circular harmonics representation. Suitable spatial source models, like point source or plane wave propagation, allow a closed-form solution of this transformation. However, it is also possible to prescribe complex wave fields as desired wave field. The transformed signals are then pre-filtered by the room compensation filters $\tilde{\mathbf{C}}$. Transformation $\mathcal{T}_2$ computes suitable loudspeaker signals from the pre-filtered transformed signal components. Wave field extrapolation, as given by Eq. (7), can be used for this purpose. Block $\mathcal{T}_3$ transforms the microphone array signals $\mathbf{l}$ into their circular harmonics representation $\tilde{\mathbf{l}}$. A suitable transformation for circular microphone arrays will be introduced in Section 6. By using only the incoming part $\breve{P}^{(1)}$ of the recorded wave field sources inside the array are omitted for room compensation purposes.

Please note, that Figure 4 also includes traditional multi-point equalization methods. In this case the transformation $\mathcal{T}_2$ and $\mathcal{T}_3$ will be diagonal matrices, and transformation $\mathcal{T}_1$ will generate the loudspeaker signals accordingly to the theory of WFS.

5. CIRCULAR LOUDSPEAKER ARRAYS

In the following we will consider the special case of circular loudspeaker arrays for WFS. As shown in Section 3.1 arbitrary wave fields can be decomposed into plane waves. Since circular arrays are symmetric with respect to their center it is sufficient to derive their characteristics for one particular incidence angle of the plane wave. The characteristics for arbitrary wave fields can then be derived from the presented results.

The theory of WFS, as presented so far, states that the wave field within the loudspeaker array can be reproduced by surrounding the listening area with

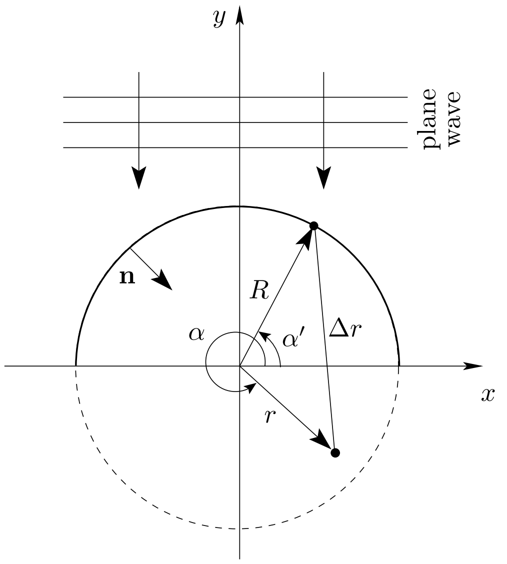


Fig. 5: Parameters used to describe the wave field reproduced by a circular WFS system.

acoustic monopoles. The strength of the monopoles is given by the directional gradient of the pressure in normal direction to the surface. Additionally, the loudspeaker selection criterion discussed in Section 2.4 has to be take into account. Specialization of the Kirchhoff-Helmholtz integral (1) to monopole secondary sources and the circular geometry allows to calculate the wave field within a circular loudspeaker array. Further specialization to the desired reproduction of one particular plane wave yields for the reproduced wave field $P_{\mathrm{r}}(\alpha, r, \omega)$

$$P_{\mathrm{r}}(\alpha, r, \omega) = \int_{\alpha_0-\pi/2}^{\alpha_0+\pi/2} \frac{\partial}{\partial \mathbf{n}} P_{\mathrm{pw}}(\alpha', R, \omega)\, \frac{e^{-jk\Delta r}}{\Delta r}\, R\, d\alpha' \tag{9}$$

where α_0 denotes the incidence angle of the plane wave and R the radius of the loudspeaker array. Figure 5 illustrates the parameters used in the integral above. Please note that $\Delta r = \Delta r(\alpha, \alpha', r, R)$ depends on the integration variables. The integration limits are chosen to fulfill the loudspeaker selection criterion mentioned. Evaluation of Eq. (9) requires to calculate the directional gradient for a plane wave. The acoustic pressure of a plane wave in polar coordinates is given as follows

$$P_{\mathrm{pw}}(\alpha, r, \omega) = e^{-jkr\cos(\alpha-\alpha_0)} \tag{10}$$

The directional gradient on the circular integration path is then derived for a plane wave as

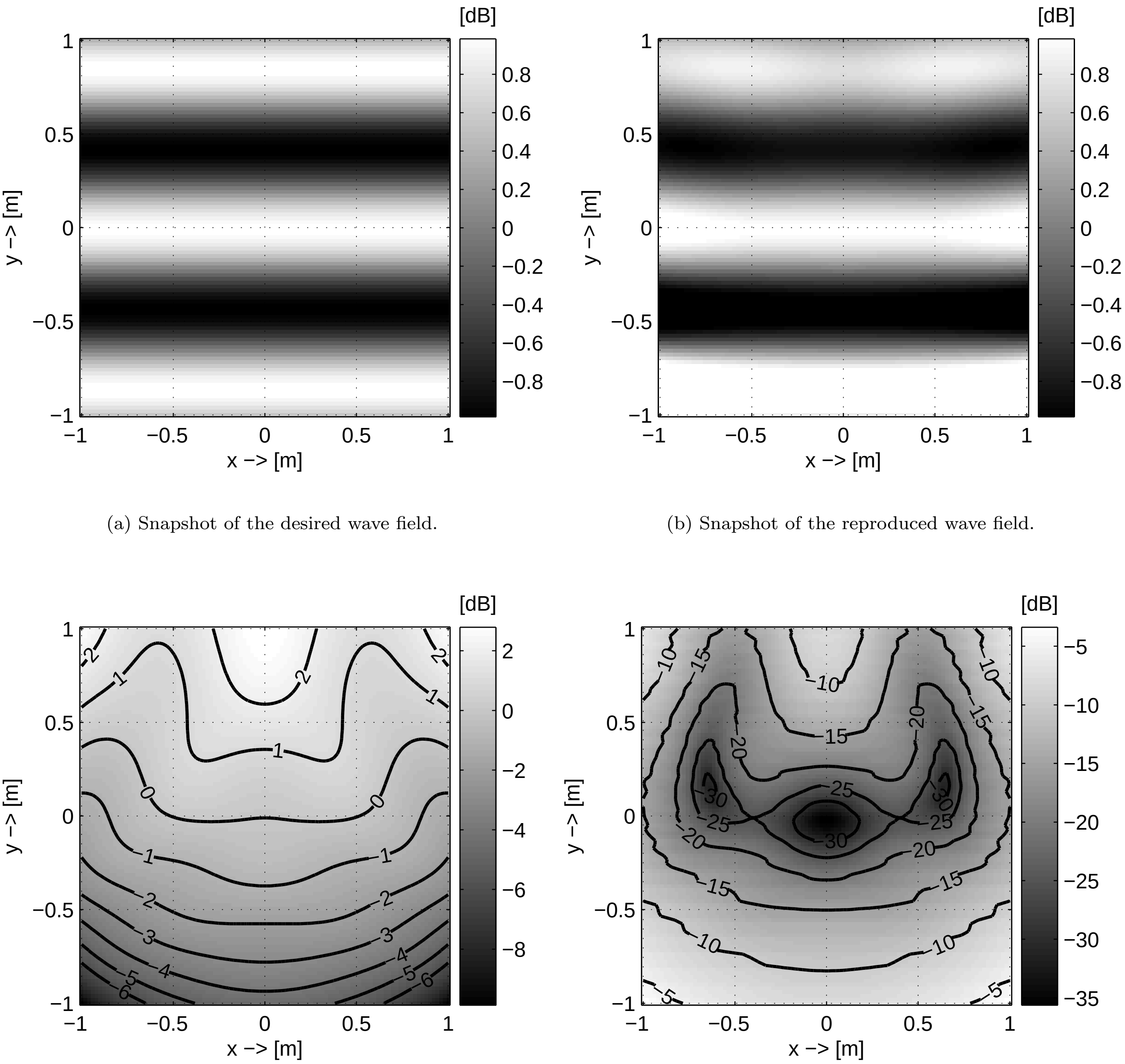
$$\frac{\partial}{\partial \mathbf{n}} P_{\mathrm{pw}}(\alpha, R, \omega) = -jkR\, \cos(\alpha - \alpha_0)\, e^{-jkR\cos(\alpha-\alpha_0)} \tag{11}$$

Introduction of above equation into Eq. (9) describes the reproduction of a plane wave using a circular WFS system.

5.1. Amplitude Errors

The following section will derive the artifacts of circular loudspeaker arrays caused by the amplitude mismatches. We will consider the reproduction of a plane with incidence angle $\alpha_0 = 90^{\mathrm{o}}$ for this purpose (see Figure 5). In the following we will present results based on numeric evaluation of Eq. (9). The radius of the loudspeaker array was chosen in accordance to the one used at our lab as $R = 1.50$ m (see Figure 3). The amplitude of the reproduced wave field was adjusted such that it equals the amplitude of the desired wave field in the center.

Figure 6 illustrates the results when reproducing a monochromatic plane wave with a frequency of $f = 400$ Hz. Figure 6(a) shows a snapshot of the desired wave field: a plane wave with incidence angle $\alpha_0 = 90^{\mathrm{o}}$ and a frequency of $f = 400$ Hz. Figure 6(b) shows a snapshot of the wave field reproduced by the circular WFS system. On first sight, the circular system described by Eq. (9) seems to be capable of reproducing a plane wave without major artifacts. However, there are some slight deviations in the amplitude visible. Figure 6(c) shows the amplitude of the reproduced plane wave. The equi-amplitude contours illustrate the amplitude variations. For the region shown the overall amplitude variation is about 8 dB. Figure 6(d) shows the averaged error between the reproduced wave field $P_{\mathrm{r}}(\alpha, r, \omega)$ and the desired wave field $P_{\mathrm{pw}}(\alpha, r, \omega)$. The error was averaged over one signal period to eliminate numerical artifacts. The error is small in the vicinity of the center due to the amplitude adjustment described above. As predicted by Figure 6(c), the error is smaller above the center. This is due to the fact that the loudspeakers above the center are used for the reproduction. Figure 6(d) shows the maximum achievable position dependent suppression that can be reached for the compensation of a plane wave by destructive interference.



(a) Snapshot of the desired wave field.

(b) Snapshot of the reproduced wave field.

(c) Amplitude of the reproduced plane wave.

(d) Averaged error between the desired and the reproduced wave field.

Fig. 6: Results when reproducing a monochromatic plane wave with an circular WFS system. The desired plane wave has an frequency of $f = 400$ Hz and an incidence angle of $\alpha_0 = 90^{\mathrm{o}}$. The radius of the simulated WFS system is $R = 1.50$ m.

5.2. Suppression of Elevated Reflections

In the previous section it was shown that a circular WFS system is capable to reproducing a desired plane wave with acceptable deviations. Thus, it can be used for listening room compensation by destructive interference. But a two-dimensional WFS system has only control over the wave field within the reproduction plane. Reflections caused by boundaries out of that plane will result in elevated contributions with respect to the reproduction plane. In the following we will show some results for the suppression of elevated plane waves.

The pressure field of a elevated plane wave in the reproduction plane ($z = 0$) is given as follows

$$P_{\mathrm{pw},\varphi_0}(\alpha, r, \omega) = e^{-jkr\cos(\varphi_0)\cos(\alpha-\alpha_0)} \qquad (12)$$

where φ_0 denotes the elevation angle. The circular WFS system was numerically simulated by introducing the directional gradient of the equation above into Eq. (9). Figure 7 shows the suppression of the incident wave field by the circular WFS system for different elevation angles of the incident plane wave. Figure 7(a) with an elevation angle $\varphi_0 = 0^{\mathrm{o}}$ is shown for reference. It is equal to Figure 6(d). As expected, an increasing elevation angle lowers the suppression of the incident field archived by the WFS system.

6. CIRCULAR MICROPHONE ARRAYS

In the following we will consider the case of a circular microphone array for WFA. Circular microphone arrays have many favorable properties, e.g. their characteristics are independent of the direction. Due to the underlying geometry it is convenient to use the circular harmonics (7) to represent the wave field. It remains then to calculate the circular harmonics expansion coefficients.

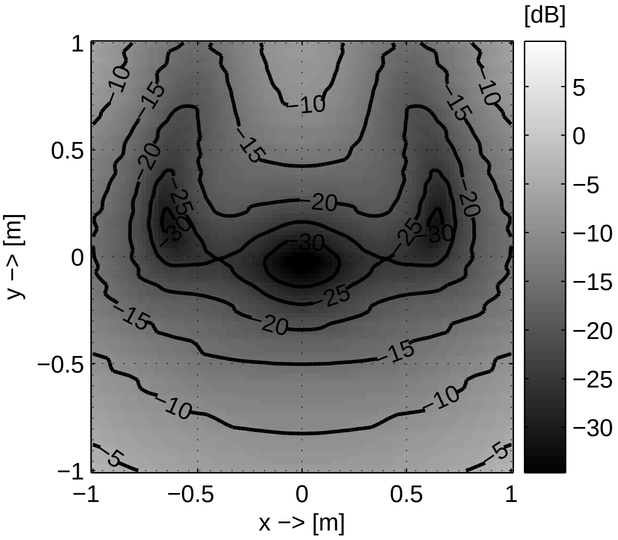
In the following we will shortly review the calculation of the cylindrical harmonics expansion coefficients as given by [16] for a circular array. Figure 8 shows a block diagram illustrating the algorithm. The starting point are acoustic pressure P and velocity measurements V_n on a circle with radius R. The acoustic velocity is measured in radial direction. The microphone position on the circle is denoted by the angle α. The first step is to calculate the Fourier series (FS) coefficients $\mathring{P}$ and $\mathring{V}_n$ of the microphone signals P and V_n. The discrete angular frequency is denoted as ν. The incoming and outgoing cylindrical harmonics expansion coefficients $\breve{P}^{(1),(2)}$ are then derived by a two-dimensional filtering operation with the filter $\mathbf{M}$. These can then be used to extrapolate the field to arbitrary positions using Eq. (7).

6.1. Amplitude Errors due to Extrapolation

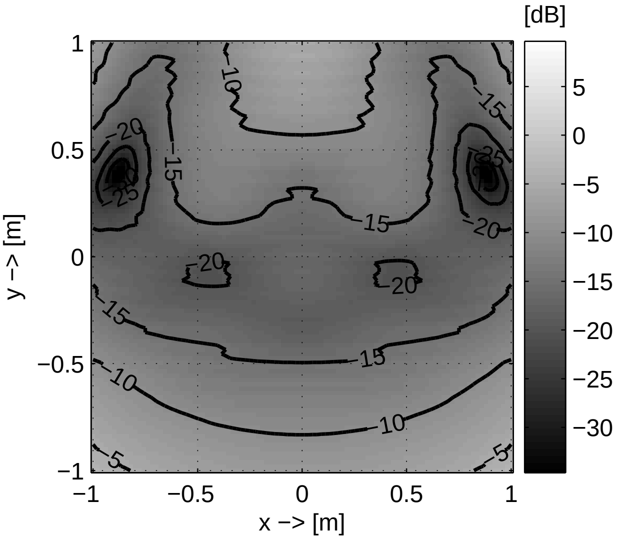
The following section will derive the artifacts when using circular harmonics for extrapolation. The Hankel functions used as basis for the decomposition exhibit a far-field amplitude decay which is inverse proportional to the square root of the extrapolation radius. Due to this property of the Hankel functions, the extrapolation of the wave field of a point source using Eq. (7) will exhibit amplitude errors. Due to the radial symmetry of the circular harmonics expansion, these amplitude errors will be radially symmetric also. Thus, it is sufficient to analyze the error for one direction α only. In the following we will present simulations based on numerical evaluation of the algorithm depicted by Figure 8. We analyzed the field of a point source, the extrapolation was done using Eq. (7).

Figure 9 illustrates the results for a point source located at a distance $d = 3$ m. Figure 9(a) shows the amplitude decays of the desired point source and the extrapolated field. The amplitudes were normalized to the radius of the array. The deviation from the desired decay of a point source after extrapolation of the field is clearly visible. Figure 9(b) shows the amplitude error between the point source and the extrapolated wave field.

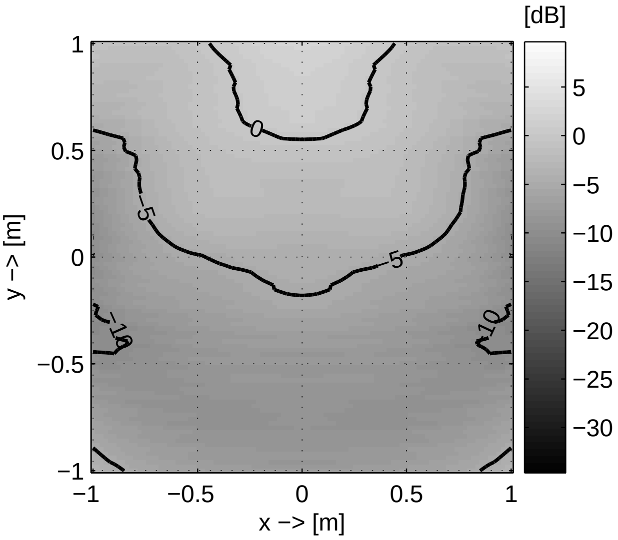
Point sources are widely used approximations for real-world sources. It would be desirable to derive an two-dimensional extrapolation technique which is capable of correctly extrapolating the wave field of a point source. Additionally, the mirror image model indicates that typical room response of a point source can be understood as a combination of primary point source and their reflections. These reflections are again modeled as point sources. In principle it is possible to modify the presented decomposition and extrapolation technique to fulfill above requirements. A drawback of such a modification would be then that the extrapolation of plane waves would exhibit amplitude errors (this effect was already discussed for WFS). One possibility for modification of Eq. (7) could be to use spherical Hankel functions instead of the Hankel functions. Another possibility proposed by [16] is to modify measured



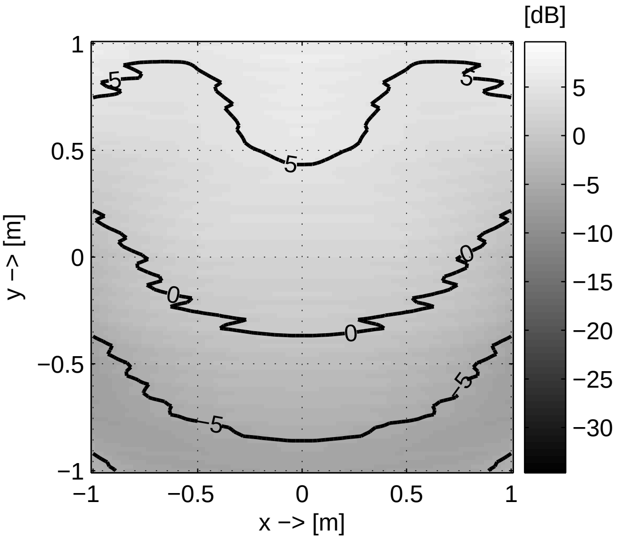
(a) Elevation angle $\varphi_0 = 0^{\mathrm{o}}$



(b) Elevation angle $\varphi_0 = 10^{\mathrm{o}}$



(c) Elevation angle $\varphi_0 = 20^{\mathrm{o}}$



(d) Elevation angle $\varphi_0 = 30^{\mathrm{o}}$

Fig. 7: Average error between the desired wave field and the reproduced for a plane wave with incidence angle $\alpha_0 = 90^{\mathrm{o}}$ and varying elevation angles φ_0.

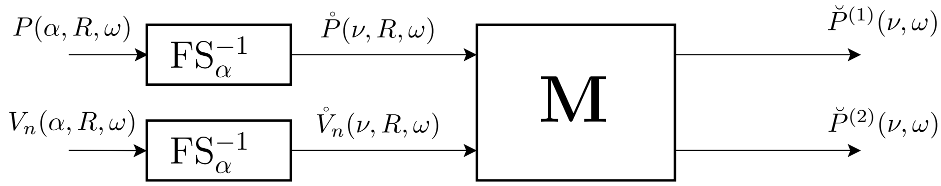
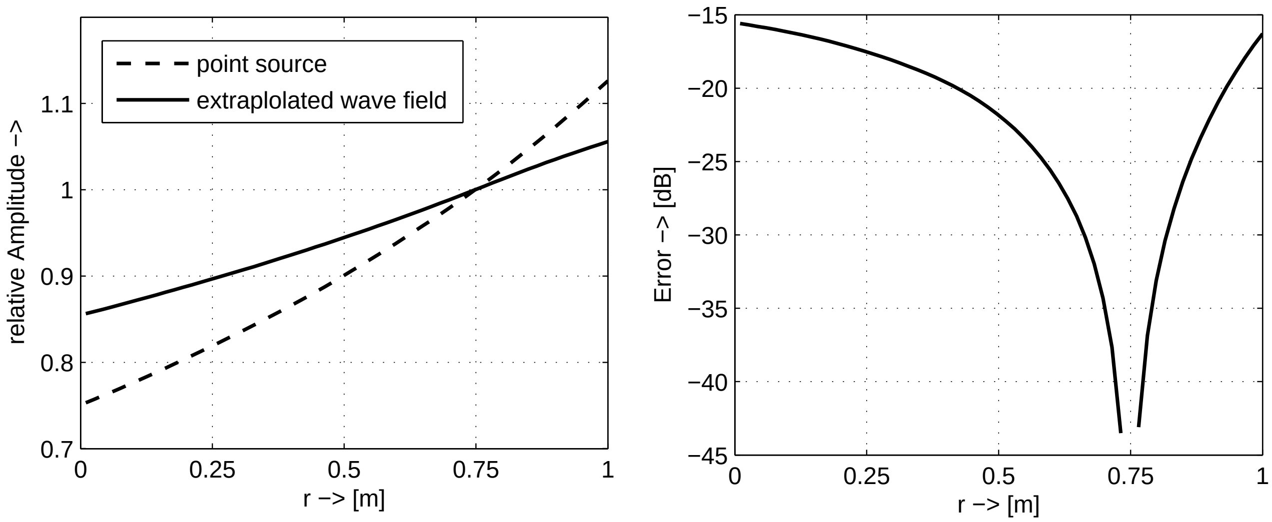


Fig. 8: Block diagram of the cylindrical harmonics decomposition for a circular microphone array.



(a) Amplitude of a point source and its extrapolated field.

(b) Amplitude error.

Fig. 9: Amplitude decay of a point source compared to the decay of the extrapolation of its measured field. The point source was located at a distance of $d = 3$ m, the radius of the array was $R = 0.75$ m.

impulse responses.

6.2. Analysis of Elevated Reflections

It was stated before, that two-dimensional analysis techniques have limited capabilities in the analysis of three-dimensional wave fields. This section will derive some results illustrating this drawback. The decomposition into plane waves can be understood as the beam-pattern in the context of microphone array analysis. The beam-pattern typically illustrates the response of a microphone array to an incident plane wave [17]. In the following we will simulate the array response for a plane wave with incidence angle $\alpha_0 = 180^o$ and varying elevation angle.

Figure 10 shows the frequency response of the array to a Dirac shaped plane wave with varying elevation angle. It can be seen that with increasing elevation angle the directionality of the array decreases. In the extreme case of Figure 10(d) (elevation angle $\varphi_0 = 90^o$) the microphone array exhibits no directionality at all. It can be concluded from the presented results that elevated plane waves interfere into all components of the decomposed field. Similar results have been reported by [18]. The presented results are also valid for other types of elevated sources, since arbitrary wave fields can be expressed as superposition of plane waves.

7. CONSEQUENCES FOR ROOM COMPENSATION USING CIRCULAR ARRAYS

The artifacts analyzed in the previous sections have consequences on the room compensation algorithm introduced in Section 4. The limitation to two-dimensional analysis and reproduction of the wave field limits the achievable performance of active listening room compensation. The following section will discuss the consequences for room compensation using circular WFA and WFS arrays. However, the presented results can be adapted quite easily to other geometries.

7.1. Amplitude Errors

It was shown in Section 5.1 that the amplitude errors of a two-dimensional WFS system pose limits on the achievable suppression of listening room reflections. The performance will depend on the listener position. The presented results reveal that a suppression of 15 dB can be reached within a region of about 1×1 m in the center of the listening area. The compensation filters of the proposed adaptive listening room compensation system are adapted such that the error between the desired and the reproduced wave field is minimized at the microphone positions (see Figure 4). As a consequence the amplitude errors will also be minimized at these positions. Thus, the results presented in Section 5.1 will scale down to the radius of the microphone array. An explicit correction of the amplitude errors in an adaptive system is thus not necessary. However, it may increase the performance of the adaptive algorithm.

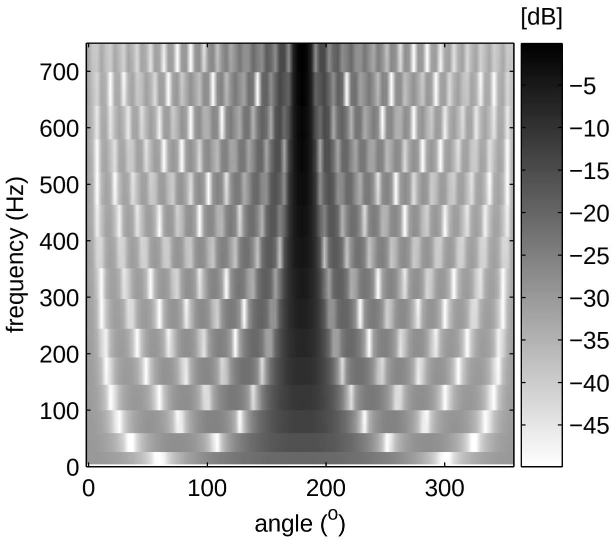
7.2. Elevated Reflections

A two-dimensional WFA and WFS system has only limited capabilities in the suppression and analysis of elevated reflections. For WFA it has been shown in Section 6.2 that elevated reflections interfere into all components of the decomposed wave field. These contributions get additionally more dominant with increasing elevation angle. As a consequence to these undesired elevated contributions, the compensation filters include contributions which only lead to a suppression of elevated listening room reflections at the microphone positions. At all other positions within the listening area these contributions will likely produce artifacts. Thus, it would be of great benefit if these elevated contributions would be suppressed by the WFA algorithm. On the reproduction side not much can be done to improve the suppression of elevated reflections when using a two-dimensional WFS system. A proper damping of the listening room at the ceiling and the floor in order to avoid elevated reflections is mandatory.

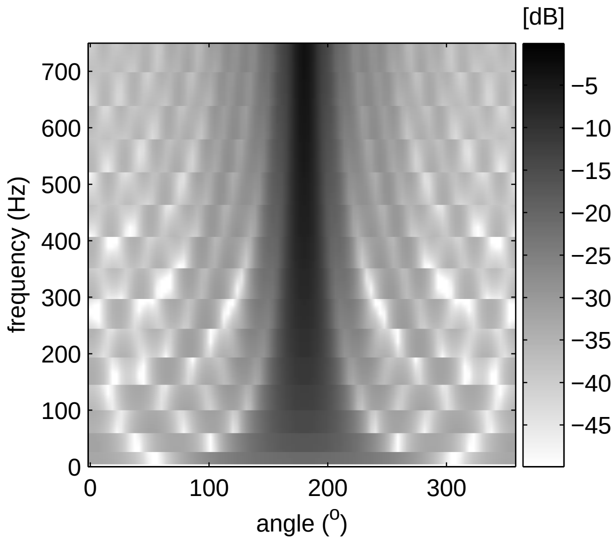
8. CONCLUSIONS

This paper presented an overview over the limiting effects of listening room compensation using WFS and WFA. A special focus was set on the analysis of the limiting artifacts for circular loudspeaker and microphone arrays. The results revealed that especially elevated reflections pose limits on the achievable performance of active room compensation.

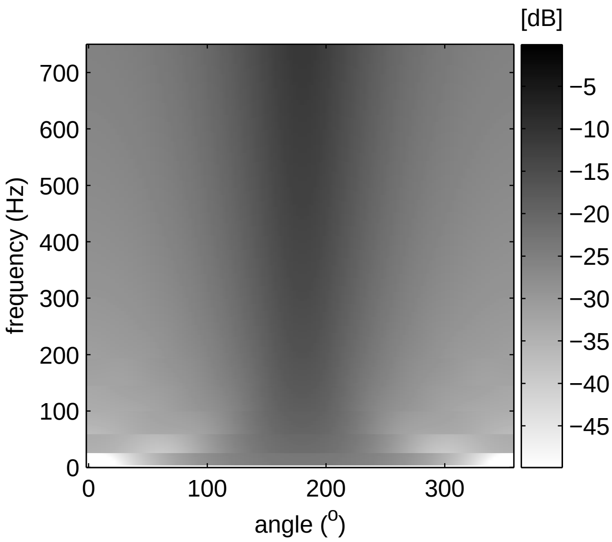
Besides the discussed artifacts, WFS and WFA will exhibit additionally artifacts. We only want to mention two here: (1) the derivation of the Kirchhoff-Helmholtz integral assumes free-field propagation from the surface to any point within the volume. In general this will not be fulfilled for WFS and WFA systems since the listeners reside in the listening area and disturb the free-field propagation. (2) The results presented in this paper where derived assuming



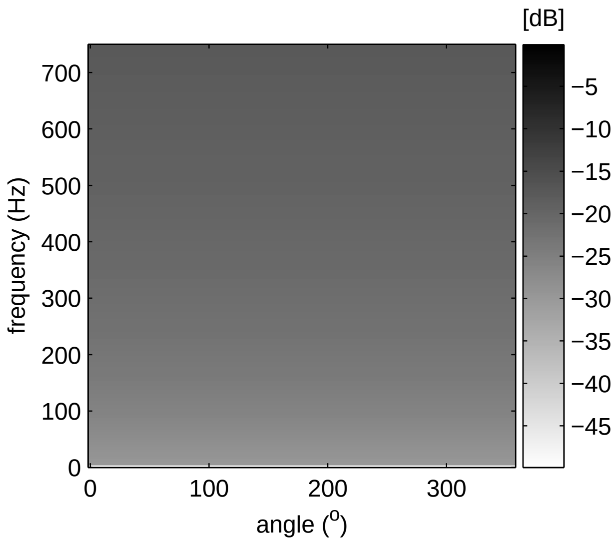
(a) Elevation angle $\varphi_0 = 0^o$



(b) Elevation angle $\varphi_0 = 30^o$



(c) Elevation angle $\varphi_0 = 60^o$



(d) Elevation angle $\varphi_0 = 90^o$

Fig. 10: Plane wave decomposition (beam-pattern) of the circular WFA array for a plane wave with an incidence angle of $\alpha_0 = 180^o$. The plots show the response for different elevation angles φ_0 of the plane wave. The radius of the array was $R = 0.75$ m.

that the listeners ears are leveled with the reproduction system. However, not all listeners will have the same height. The compensation of reflections for planes above or under the reproduction plane will not have the same performance as within this plane.

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