

NON-LINEAR CONVOLUTION: A NEW APPROACH FOR THE AURALIZATION OF DISTORTING SYSTEMS

Angelo Farina, Alberto Bellini and Enrico Armelloni

Industrial Engineering Dept., University of Parma, Via delle Scienze 181/A
Parma, 43100 ITALY – **[HTTP://pcfarina.eng.unipr.it](http://pcfarina.eng.unipr.it)**

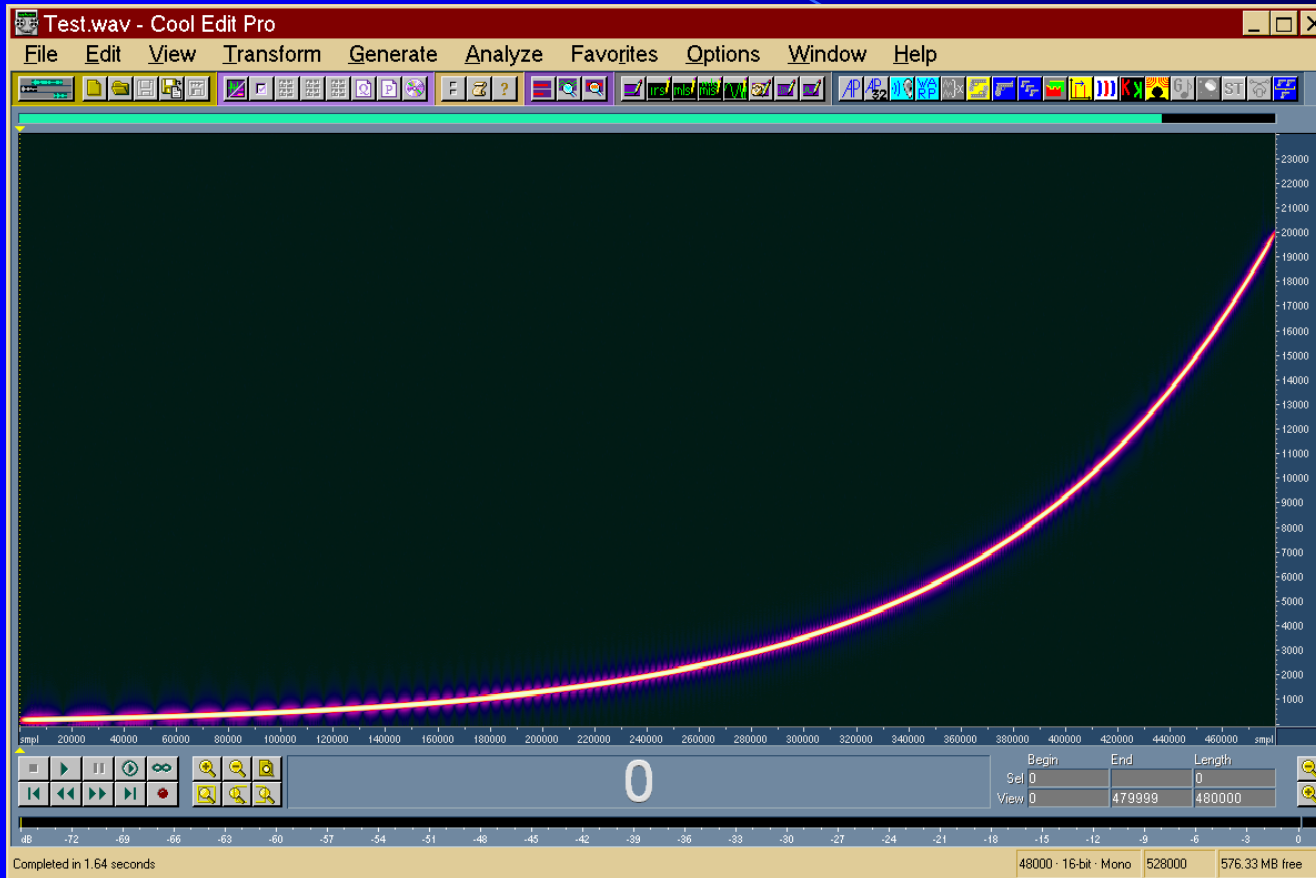
Goals for Auralization

- Transform the results of objective electroacoustics measurements to audible sound samples suitable for listening tests
- Traditional auralization is based on linear convolution: this does not replicates faithfully the nonlinear behaviour of most transducers
- The new method presented here overcomes to this strong limitation, providing a simplified treatment of memory-less distortion

Methods

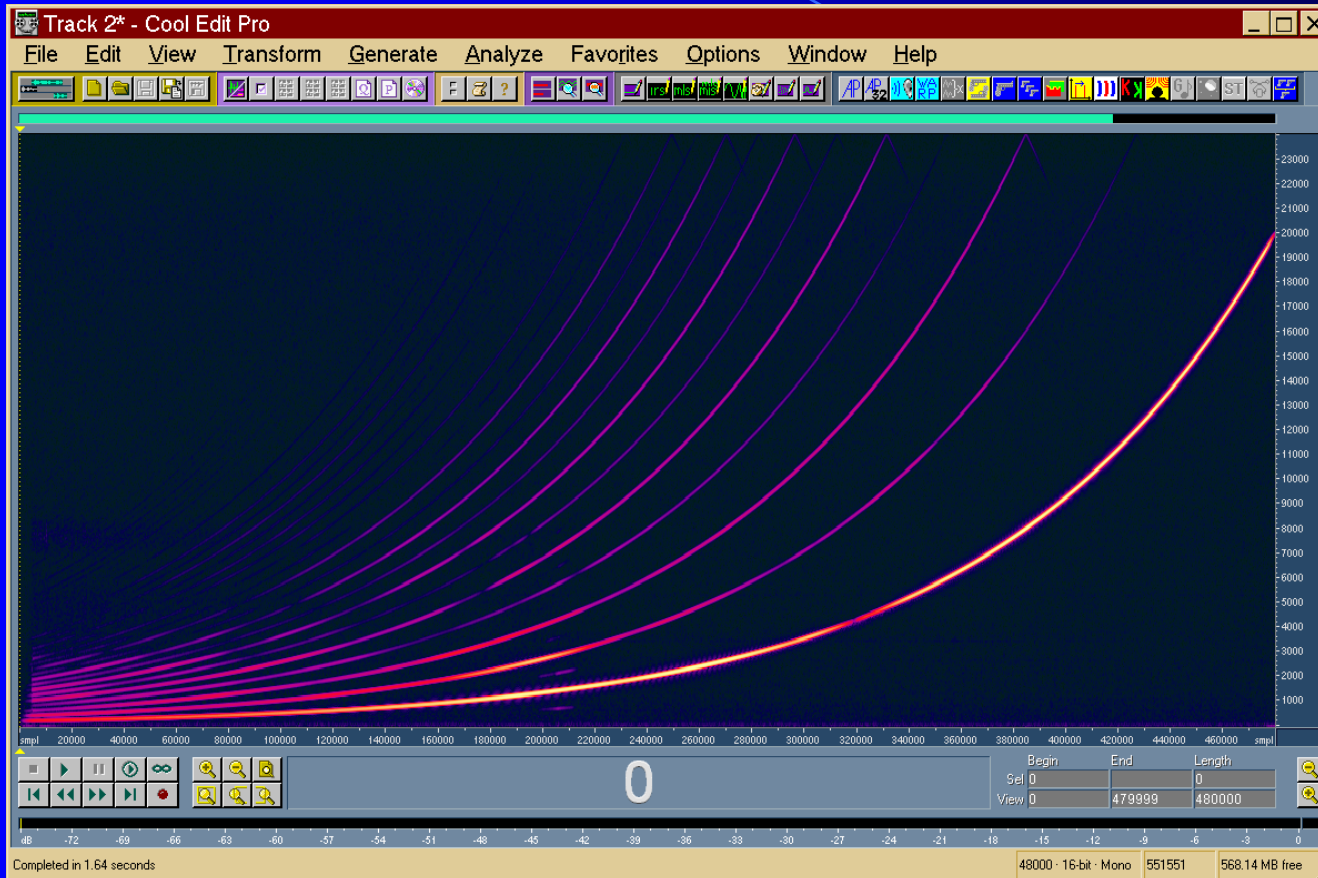
- We start from a measurement of the system based on exponential sine sweep (Farina, 108th AES, Paris 2000)
- Diagonal Volterra kernels are obtained by post-processing the measurement results
- These kernels are employed as FIR filters in a multiple-order convolution process (original signal, its square, its cube, and so on are convolved separately and the result is summed)

Exponential sweep measurement



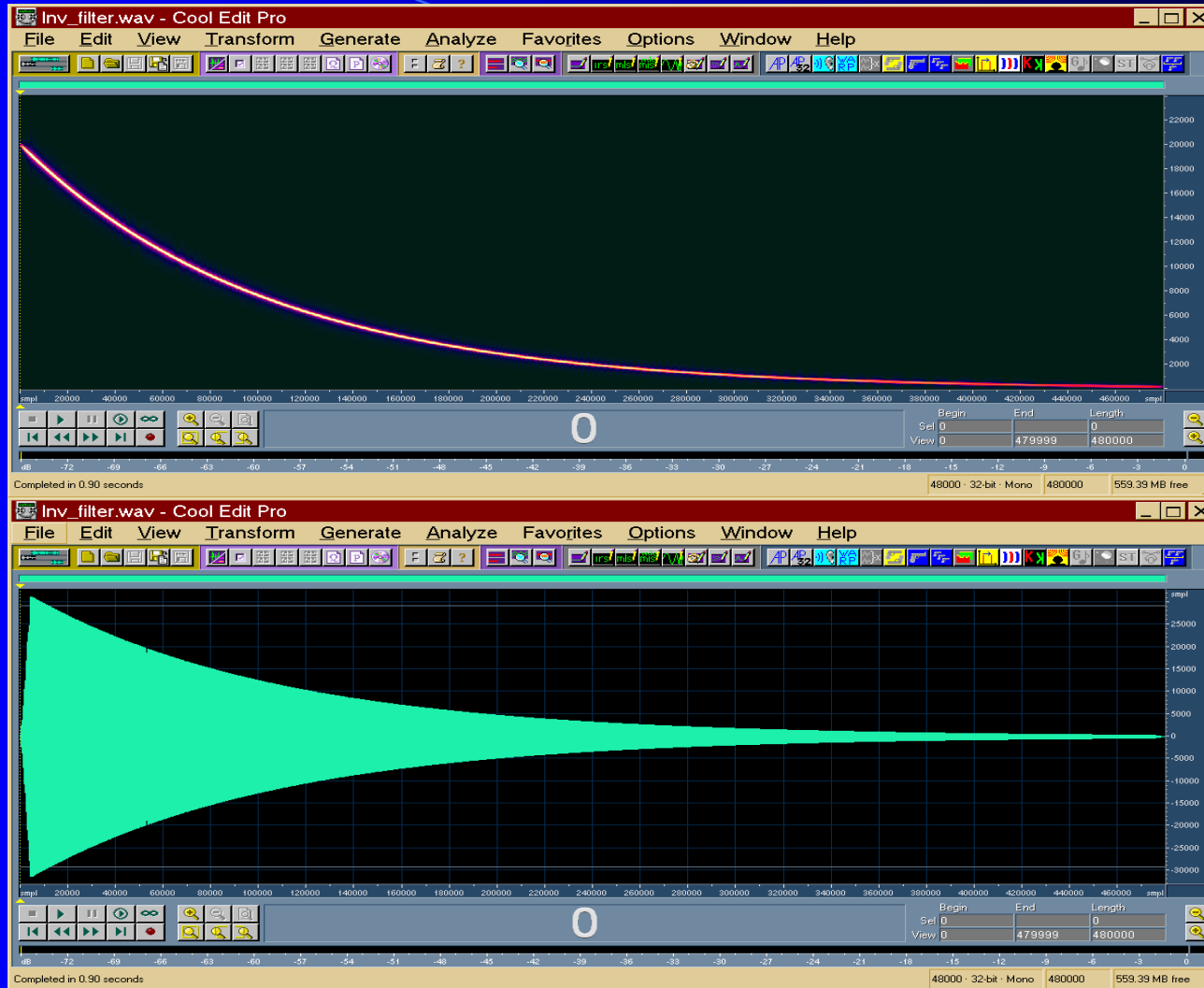
- The excitation signal is a sine sweep with constant amplitude and exponentially-increasing frequency

Raw response of the system



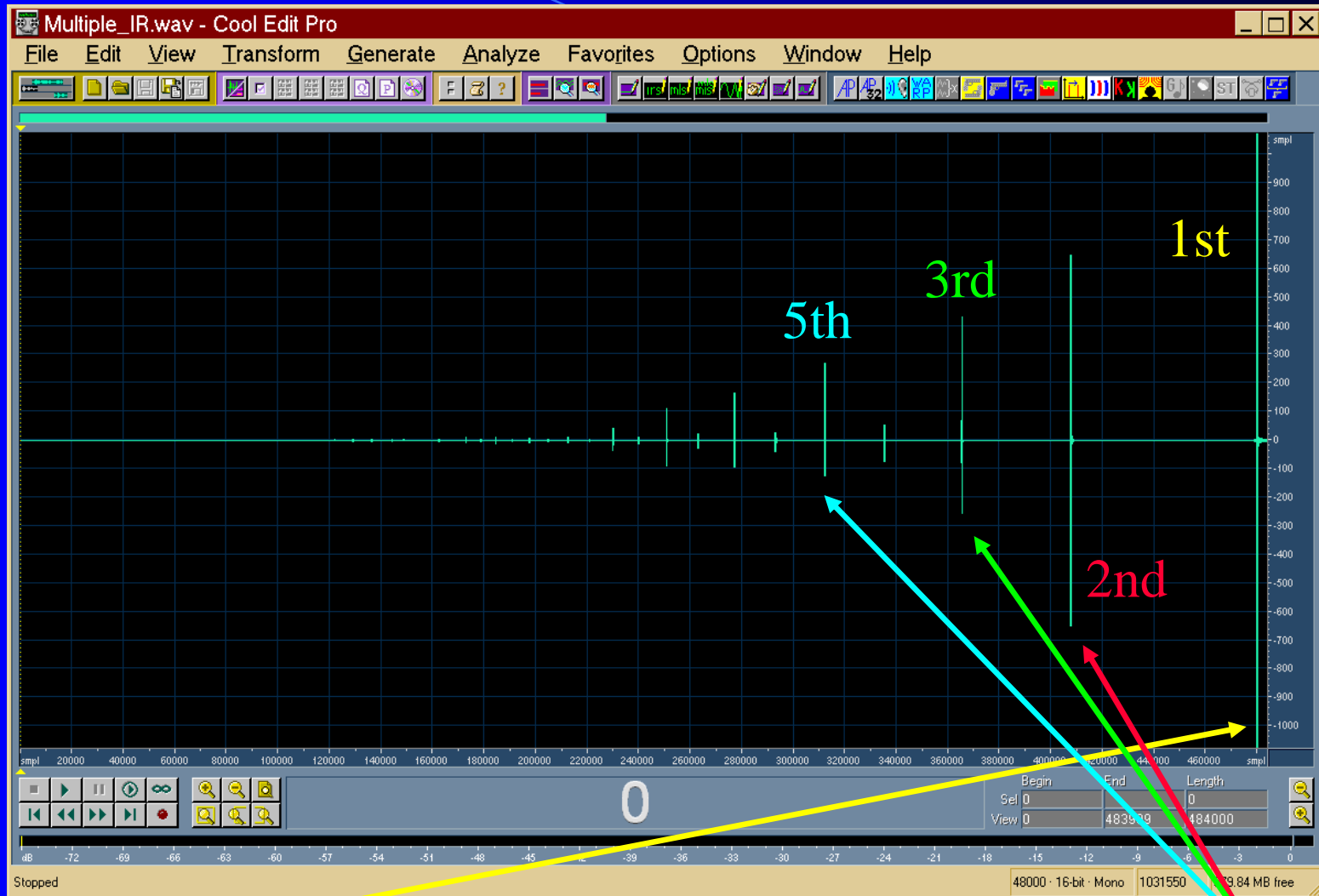
Many harmonic orders do appear as colour stripes

Deconvolution of system's impulse response



The deconvolution is obtained by convolving the raw response with a suitable inverse filter

Multiple impulse response obtained



The **last peak** is the linear impulse response, the **preceding ones** are the harmonic distortion orders

Auralization by linear convolution



Convoluting a suitable sound sample with the linear IR, the frequency response and temporal transient effects of the system can be simulated properly

What's missing in linear convolution ?

- No harmonic distortion, nor other nonlinear effects are being reproduced.
- From a perceptual point of view, the sound is judged “cold” and “innatural”
- A comparative test between a strongly nonlinear device and an almost linear one does not reveal any audible difference, because the nonlinear behavior is removed for both

Theory of nonlinear convolution

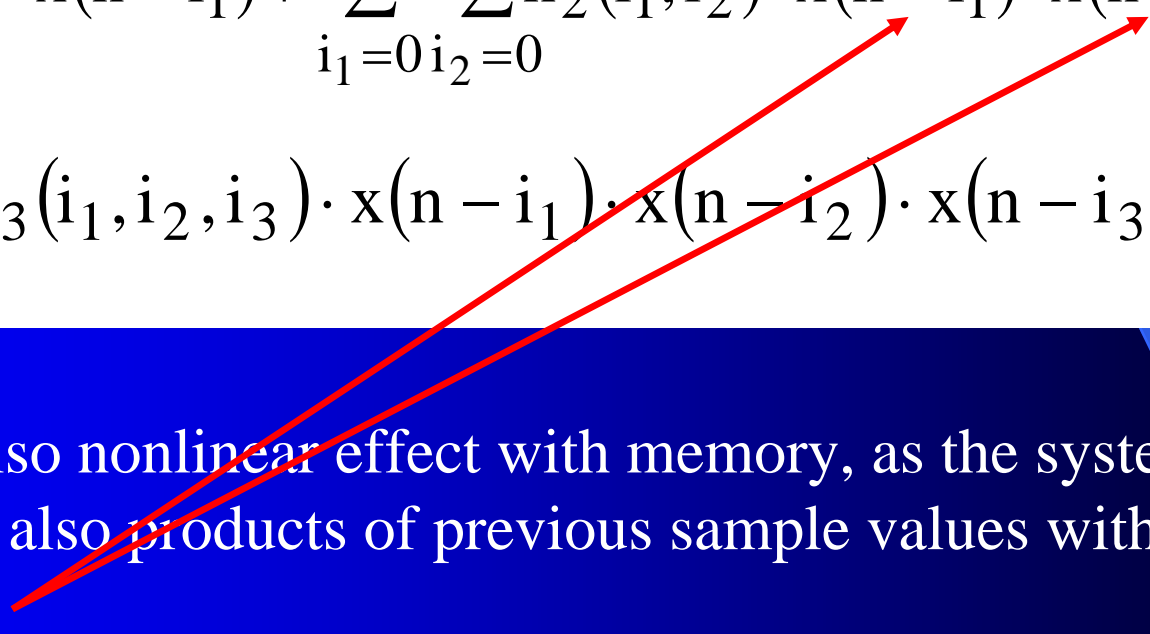
- The basic approach is to convolve separately, and then add the result, the linear IR, the second order IR, the third order IR, and so on.
- Each order IR is convolved with the input signal raised at the corresponding power:

$$y(n) = \sum_{i=0}^{M-1} h_1(i) \cdot x(n-i) + \sum_{i=0}^{M-1} h_2(i) \cdot x^2(n-i) + \sum_{i=0}^{M-1} h_3(i) \cdot x^3(n-i) + \dots$$

The problem is that the required multiple IRs **are not** the results of the measurements: they are instead the diagonal terms of Volterra kernels

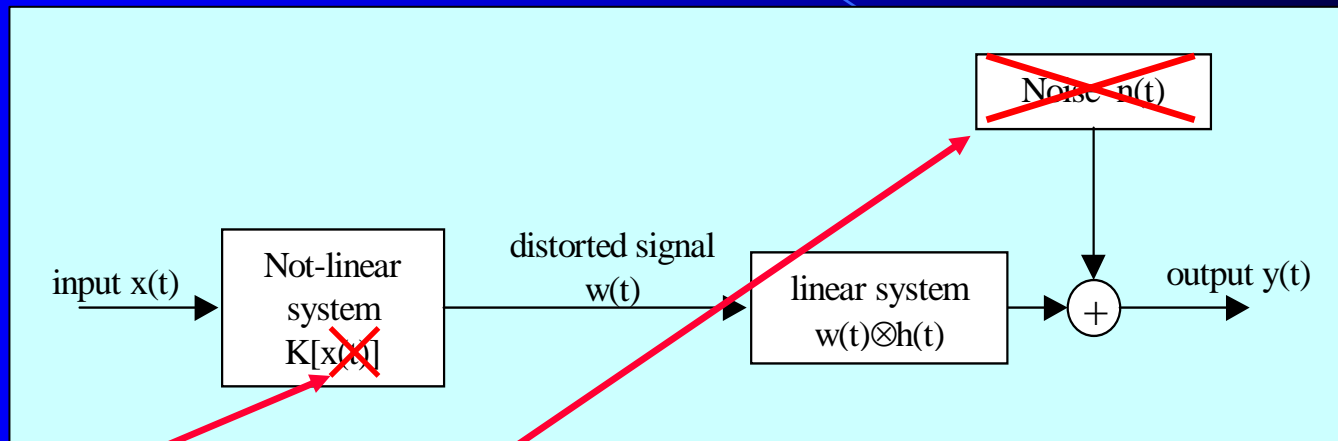
Volterra kernels and simplification

- The general Volterra series expansion is defined as:

$$y(n) = \sum_{i_1=0}^{M-1} h_1(i_1) \cdot x(n-i_1) + \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} h_2(i_1, i_2) \cdot x(n-i_1) \cdot x(n-i_2) + \\ + \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} \sum_{i_3=0}^{M-1} h_3(i_1, i_2, i_3) \cdot x(n-i_1) \cdot x(n-i_2) \cdot x(n-i_3) + \dots$$


This explains also nonlinear effect with memory, as the system output contains also products of previous sample values with different delays

Memoryless distortion followed by a linear system with memory



- The first nonlinear system is assumed to be memory-less, so only the diagonal terms of the Volterra kernels need to be taken into account.
- Furthermore, we neglect the noise, which is efficiently rejected by the sine sweep measurement method.

Volterra kernels from the measurement results

The measured multiple IRs h' can be defined as:

$$y(t) = h'_1 \otimes \sin[\omega_{\text{var}}] + h'_2 \otimes \sin[2 \cdot \omega_{\text{var}}] + h'_3 \otimes \sin[3 \cdot \omega_{\text{var}}] + \dots$$

We need to relate them to the simplified Volterra kernels h :

$$y(t) = h_1 \otimes \sin[\omega_{\text{var}}] + h_2 \otimes \sin^2[\omega_{\text{var}}] + h_3 \otimes \sin^3[\omega_{\text{var}}] + \dots$$

Trigonometry can be used to expand the powers of the sinusoidal terms:

$$\sin^2(\omega \cdot \tau) = \frac{1}{2} - \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot \tau)$$

$$\sin^3(\omega \cdot \tau) = \frac{3}{4} \cdot \sin(\omega \cdot \tau) - \frac{1}{4} \cdot \sin(3 \cdot \omega \cdot \tau)$$

$$\sin^4(\omega \cdot \tau) = \frac{3}{8} - \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot \tau) + \frac{1}{8} \cdot \cos(4 \cdot \omega \cdot \tau)$$

$$\sin^5(\omega \cdot \tau) = \frac{5}{8} \cdot \sin(\omega \cdot \tau) - \frac{5}{16} \cdot \sin(3 \cdot \omega \cdot \tau) + \frac{1}{16} \cdot \sin(5 \cdot \omega \cdot \tau)$$

Finding the connection point

Going to frequency domain by taking the FFT, the first equation becomes:

$$Y(\omega) = \bar{H}'_1[\omega] \cdot X[\omega] + \bar{H}'_2[\omega] \cdot X[\omega/2] + \bar{H}'_3[\omega] \cdot X[\omega/3] + \dots$$

Doing the same in the second equation, and substituting the trigonometric expressions for power of sines, we get:

$$Y(\omega) = \left[\bar{H}_1 + \frac{3}{4} \cdot \bar{H}_3 + \frac{5}{8} \cdot \bar{H}_5 \right] \cdot X[\omega] + \left[-\frac{1}{2} \cdot \bar{H}_2 - \frac{1}{2} \cdot \bar{H}_4 \right] \cdot j \cdot X[\omega/2] + \left[-\frac{1}{4} \cdot \bar{H}_3 - \frac{5}{16} \cdot \bar{H}_5 \right] \cdot X[\omega/3] + \frac{1}{8} \cdot \bar{H}_4 \cdot j \cdot X[\omega/4] + \frac{1}{16} \cdot \bar{H}_5 \cdot X[\omega/5] + \dots$$

The terms in square brackets have to be equal to the corresponding measured transfer functions H' of the first equation

Solution

- Thus we obtain a linear equation system:

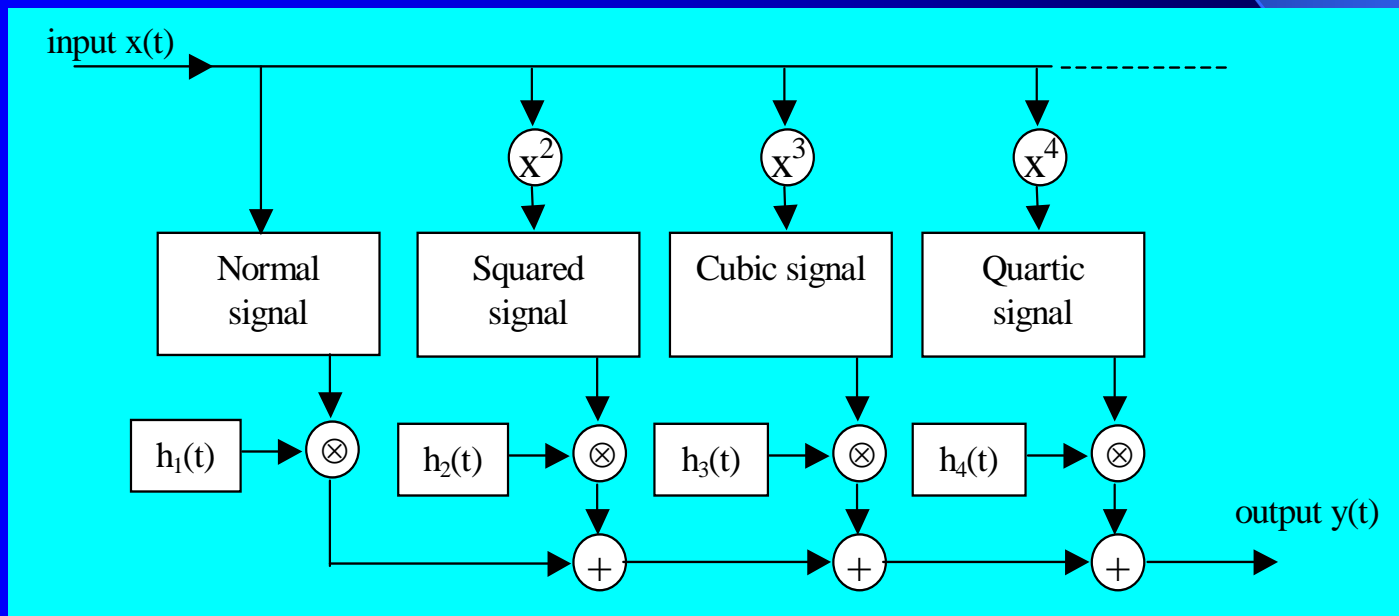
$$\begin{cases} \bar{H}'_1 = \bar{H}_1 + \frac{3}{4} \cdot \bar{H}_3 + \frac{5}{8} \cdot \bar{H}_5 \\ \bar{H}'_2 = -j \cdot \frac{1}{2} \cdot [\bar{H}_2 + \bar{H}_4] \\ \bar{H}'_3 = -\frac{1}{4} \cdot \bar{H}_3 - \frac{5}{16} \cdot \bar{H}_5 \\ \bar{H}'_4 = j \cdot \frac{1}{8} \cdot \bar{H}_4 \\ \bar{H}'_5 = \frac{1}{16} \cdot \bar{H}_5 \end{cases}$$

We can easily solve it, obtaining the required Volterra kernels as a function of the measured multiple-order IRs:

$$\begin{cases} \bar{H}_1 = \bar{H}'_1 + 3 \cdot \bar{H}'_3 + 5 \cdot \bar{H}'_5 \\ \bar{H}_2 = 2 \cdot j \cdot \bar{H}'_2 + 8 \cdot j \cdot \bar{H}'_4 \\ \bar{H}_3 = -4 \cdot \bar{H}'_3 - 20 \cdot \bar{H}'_5 \\ \bar{H}_4 = -8 \cdot j \cdot \bar{H}'_4 \\ \bar{H}_5 = 16 \cdot \bar{H}'_5 \end{cases}$$

Non-linear convolution

As we have got the Volterra kernels already in frequency domain, we can efficiently use them in a multiple convolution algorithm implemented by overlap-and-save of the partitioned input signal:



Software implementation

Although today the algorithm is working off-line (as a mix of manual CoolEdit operations and some Matlab processing), a more efficient implementation as a CoolEdit plugin is being worked out:

Multiple Convolution with Clipboard

Stimulus	Impulse Response
N. of sweeps / measurement: 4	N. of samples for each response: 4096
Start Frequency (Hz): 40	N. of first samples to skip: 0
End Frequency (Hz): 18000	Number of Harmonic Orders: 10
Sweep duration (s or samples): 10	<input checked="" type="checkbox"/> Autoscale and remove DC component
Silence duration (s or samples): 1	User: Angelo Farina
	Reg. key: *****

OK Help Cancel

This will allow for real-time operation even with a very large number of filter coefficients

Audible evaluation of the performance

Original signal

Linear convolution



These last two were compared in a formalized blind listening test



Live recording



Non-linear multi convolution

Subjective listening test

- A/B comparison
- Live recording & non-linear auralization
- 12 selected subjects
- 4 music samples
- 9 questions
- 5-dots horizontal scale
- Simple statistical analysis of the results
- A was the live recording, B was the auralization, but the listener did not know this

The screenshot shows a software window titled "Risposte soggettive" with a red title bar. At the top, there are controls for "Brano n." (1, 2, 3, 4) and "A" (highlighted in green) and "B". Below this is a file path: "D:\Convola_top_lamiera\05RebeccaPidgeon-porta.WAV". The main area contains nine questions, each with a 5-dot horizontal scale and red double-headed arrows indicating the response range:

- Domanda 1: A & B are identical vs. A & B are quite different
- Domanda 2: A is more enveloping vs. B is more enveloping
- Domanda 3: A has better timber vs. B has better timber
- Domanda 4: A is more dry vs. B is more dry
- Domanda 5: A is more distorted vs. B is more distorted
- Domanda 6: A has more treble vs. B has more treble
- Domanda 7: A has more medium vs. B has more medium
- Domanda 8: A has more bass vs. B has more bass
- Domanda 9: A is more pleasant vs. B is more pleasant

At the bottom, there are buttons for "Precedente", "Successivo", and "Fine". An orange arrow points from a box at the bottom left to the "Successivo" button.

95% confidence intervals
of the responses

Conclusion

Statistical parameters – more advanced statistical methods would be advisable for getting more significant results

Question Number	Average score	2.67 * Std. Dev.
1 (identical-different)	1.25	0.76
3 (better timber)	3.45	1.96
5 (more distorted)	2.05	1.34
9 (more pleasant)	3.30	2.16

Final remarks

- The CoolEdit plugin is planned to be released in two months – it will be downloadable from [HTTP://www.ramsete.com/aurora](http://www.ramsete.com/aurora)
- The sound samples employed for the subjective test are available for download at [HTTP://pcangelo.eng.unipr.it/public/AES110](http://pcangelo.eng.unipr.it/public/AES110)
- The new method will be employed for realistic reproduction in a listening room of the behaviour of car sound systems